Equilibrium Liquidity and Risk Offsetting in Decentralised Markets

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Joint work with

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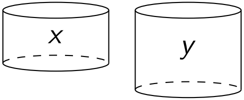
Introduction

To set the stage....

- ▷ Liquidity Pooling
  - A **pool** with assets X & Y
  - Available liquidity (**reserves**): x and y



Pool reserves

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- Two types of market participants

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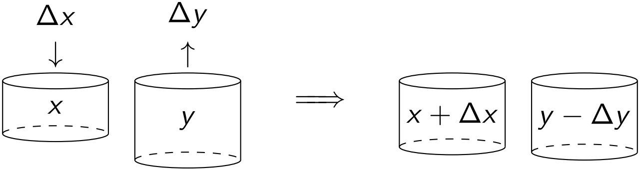
  - liquidity takers (**LTs**) **trade** with the pool.



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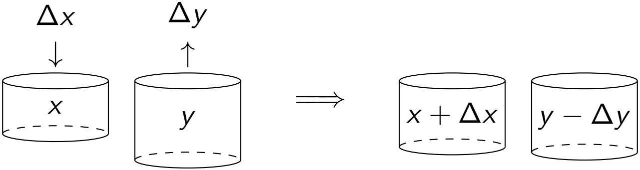
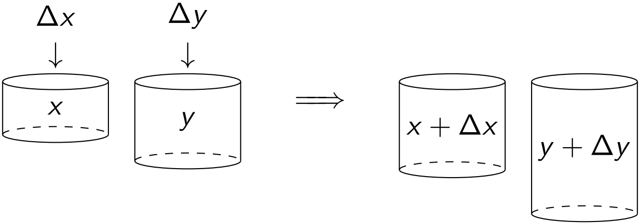


  - liquidity providers (**LPs**) **deposit** assets in the pool or **withdraw** assets from the pool



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   ▷ As risk aversion increases relative to CEX trading costs, equilibrium DEX liquidity falls, and beyond a threshold, liquidity provision may cease entirely (market shutdown).

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   - ▷ Strong signals induce the LP to withdraw liquidity because exploiting the information requires intensive and costly CEX trading
   - ▷ This results in thinner DEX markets and lower volumes for uninformed traders.

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   ▷ Higher arrival rates or lower price sensitivity of uninformed traders support deeper liquidity

   ▷ ... even in volatile markets, whereas high fundamental volatility alone can destroy liquidity when fee revenue cannot compensate for adverse selection and hedging costs.

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- **LTs and LPs**
  - LTs send (receive) a quantity $\mathbf{\Delta y}$ of Y. They receive (send) a quantity $\mathbf{\Delta x}$ of X s.t.

$$f(x, y) = f(x \mp \mathbf{\Delta x}, y \pm \mathbf{\Delta y}) = \kappa^2 \quad \leftarrow \quad \textbf{Depth}$$

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- **Level function (bonding curve)**
  - $f(x, y) = \kappa^2 \iff x = \varphi(\kappa, y)$.
  - **bonding curves** map reserves in Y to reserves in X.
  - They define price impact and execution prices.

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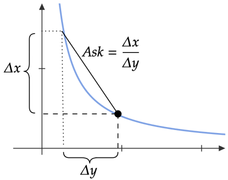
Price of liquidity: Bid/Ask for Δy

$$\text{Ask} = \frac{\Delta x}{\Delta y} = \frac{\varphi(y - \Delta y) - \varphi(y)}{\Delta y}$$



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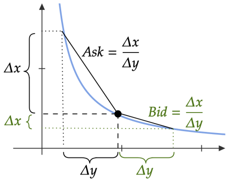

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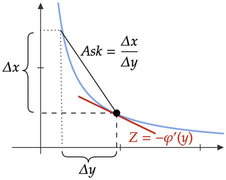

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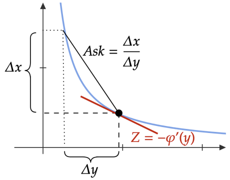

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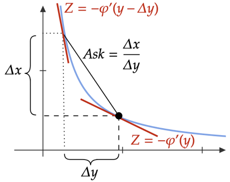

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Price impact for quantity Δy:

$$-\varphi'(y + \Delta y) \xleftarrow{\text{sell}} \underbrace{-\varphi'(y)}_{\text{marginal price}} \xrightarrow{\text{buy}} -\varphi'(y - \Delta y)$$

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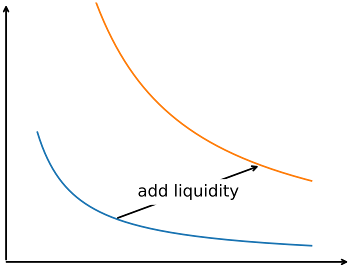

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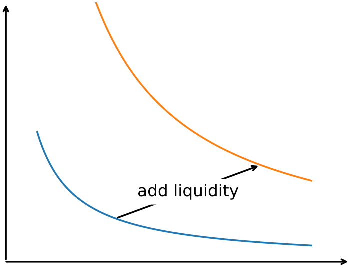

We consider a representative Liquidity provider (RLP)... what is the **"optimal"** level of liquidity to provide?

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- **Endogenous** liquidity reserves ultimately **drive DEX trading volumes**... not the other way around.
- The literature also overlooks practical realities such as CEX **trading costs**, **risk preferences**, and LPs' strategic use of **private information**.

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where $\mathbf{A} = (A_t)_{t\in[0,T]}$ is a progressively measurable process s.t. $\mathbb{E}\left[\int_0^T |A_t|^p\,\mathrm{d}t\right] < \infty$ for some $p > 2$

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- We make assumptions s.t.

$$F_t = -\partial_1 \varphi(Y_t, \kappa) \iff Y_t = h(F_t, \kappa).$$

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- The dynamics of the value of the DEX reserves in units of the reference asset X are

$$\mathrm{d}\left(X_t + Y_t\, F_t\right) = Y_t\, \mathrm{d}F_t - \underbrace{\tfrac{1}{2}\partial_{11}\varphi(h(F_t,\kappa),\kappa)\,\left(\partial_1 h(F_t,\kappa)\right)^2\,\sigma^2\,F_t^2\,\mathrm{d}t}_{\text{loss-versus-rebalancing (LVR)}}$$

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- LPs who short a replication of their DEX position corresponds to offsetting the first term ... thus exposing the LP to LVR
- **LVR must be compensated by fees**
  - When an LT buys Δy of Y they pay an additional fee of $\pi\,\Delta y\,F_t$
  - The cost per unit of Y is therefore

$$\frac{\varphi(Y_t - \Delta y,\kappa) - \varphi(Y_t,\kappa) + \pi\,\Delta y\,F_t}{\Delta y}$$

Three Stages

- The agents interact in three stages:

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- if $\mathbf{V > 0}$ and LT wishes to **buy** a quantity $\delta > 0$ of asset Y, her execution price is

$$\frac{1}{\delta}\left(\varphi(Y_t - \delta, \kappa) - \varphi(Y_t, \kappa) + \pi\,\delta\,F_t\right) \approx F_t + \pi\,F_t + \tfrac{1}{2}\delta\,\partial_{11}\varphi(Y_t, \kappa)$$

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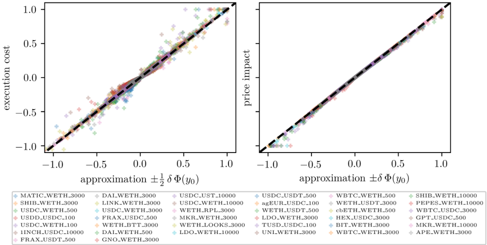
- if $\mathbf{V} < \mathbf{0}$ and LT wishes to **sell** a quantity $\delta > 0$ of asset Y, her execution price is

$$\frac{1}{\delta}\left(\varphi(Y_t,\kappa) - \varphi(Y_t+\delta,\kappa) - \pi\,\delta\,F_t\right) \approx F_t - \pi\,F_t - \tfrac{1}{2}\delta\,\partial_{11}\varphi(Y_t,\kappa)$$

Three Stages — Stage III

- At stage III — LTs arrive
  - **Arbitrageurs align** the DEX and CEX **price** — we ignore their fees
  - **noise LTs** arrive (at Poisson times) with **elastic demand**
    - arrive with **private utility** V

- Approximation is accurate for such markets...



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- Assign a utility of $(1 + V)F_t$ for holding the asset

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...optimal is

$$\delta_t^\star = F_t \, \frac{|V| - \pi}{\partial_{11}\varphi\left(Y_t, \kappa\right)}$$

- The nLTs generate **stochastic fees** for the LP, worth

$$\mathbb{E}\left[\int_0^T \pi \, \delta_t^\star \, F_t \, dN_t\right] = \mathbb{E}\left[\int_0^T \frac{\lambda \, \pi \left(v - \pi\right) F_t^2}{\partial_{11}\varphi\left(h(F_t, \kappa), \kappa\right)} \, dt\right]$$

Three Stages

- The agents interact in three stages:

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with

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- LP's **DEX reserves** in asset Y satisfies

$$\mathrm{d}Y_t = G_t\,F_t\,\mathrm{d}t + \sigma\,\partial_1 h(F_t,\kappa)\,F_t\,\mathrm{d}W_t$$

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- ▷ **LP's exposure** in the DEX has value

$$L_t^\nu := \underbrace{\int_0^t \Pi(F_u, \kappa)\, \mathrm{d}u}_{\text{fee revenue}} + \overbrace{X_t + Y_t\, S_t^\nu}^{\text{MtM liquidity value}}$$

Three Stages — Stage II

- ▷ LP trading strategy for a given κ
- ▷ LP trades continuously in the CEX and holds inventory

$$Q_t^{\nu} = Q_0 + \int_0^t \nu_s \, \mathrm{d}s$$

Three Stages — Stage II

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$$Q_t^\nu = Q_0 + \int_0^t \nu_s \, \mathrm{d}s$$

- **LP's overall criterion** is

$$J[\nu] := \mathbb{E}\left[L_T^\nu + Q_T^\nu \, S_T^\nu - \int_0^T (S_t^\nu + \eta \, \nu_t) \, \nu_t \, \mathrm{d}t - \frac{\phi}{2} \int_0^T (Q_t^\nu + Y_t)^2 \, \mathrm{d}t \right]$$

$$= \mathbb{E}\Bigg[\overbrace{(Y_T + Q_T^\nu) \; S_T^\nu}^{\text{combined CEX-DEX position}} - \overbrace{\int_0^T (S_t^\nu + \eta \, \nu_t) \; \nu_t \, \mathrm{d}t}^{\text{risk offsetting}} \\ \underbrace{- \frac{\phi}{2} \int_0^T (Q_t^\nu + Y_t)^2 \; \mathrm{d}t}_{\text{deviation penalty}} \Bigg] + const.$$

Three Stages — Stage II

Proposition

Define the symmetric bounded linear operator $\Lambda : \mathcal{A}_2 \to \mathcal{A}_2$ *by*

$$\Lambda := 2\,\eta + \beta\,(\mathfrak{I}^\top \mathfrak{L} + \mathfrak{L}^\top \mathfrak{I}) - c\,(\mathfrak{L} + \mathfrak{L}^\top) + \phi\,\mathfrak{L}^\top \mathfrak{L}$$

and $\upsilon \in \mathcal{A}_2$ *by*

$$\upsilon := \mathfrak{I}^\top (G\,F) + (c - \beta\,\mathfrak{I}^\top - \phi\,\mathfrak{L}^\top)(Y + Q_0) + \mathfrak{L}^\top (A\,F)\,.$$

Then the objective J *satisfies*

$$J[\nu] = -\tfrac{1}{2}\,\langle \Lambda\nu, \nu\rangle + \langle \upsilon, \nu\rangle.$$

where the two bounded linear operators $\mathfrak{L}\,,\mathfrak{I} : \mathcal{A}_2 \to \mathcal{A}_2$ are

$$(\mathfrak{L}\nu)_t = \int_0^t \nu_s\,\mathrm{d}s \qquad \text{and} \qquad (\mathfrak{I}\nu)_t = c\int_0^t e^{\beta(s-t)}\,\nu_s\,\mathrm{d}s\,.$$

Three Stages — Stage II

Proposition

J is Gâteaux differentiable, and its Gâteaux derivative $\mathcal{D}J[\nu]$ at $\nu \in \mathcal{A}_2$ is an element of $\mathcal{A}_2$ and

$$\begin{aligned}\mathcal{D}J[\nu]_t = -2\,\eta\,\nu_t + c\,(Y_t + Q^\nu_t) \\ &+ \mathbb{E}\left[\int_t^T (A_s\,F_s + c\,\nu_s - \beta\,I^\nu_s - \phi\,(Y_s + Q^\nu_s))\,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right] \\ &+ c\,e^{t\,\beta}\,\mathbb{E}\left[\int_t^T e^{-s\,\beta}\,(G_s\,F_s - \beta\,(Y_s + Q^\nu_s))\,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right].\end{aligned}$$

Three Stages — Stage II

Theorem (FBSDE system)

The Gâteaux derivative $\mathcal{D}J[\cdot]$ vanishes at $\nu^\star \in \mathcal{A}_2$ if and only if $\nu^\star$ solves the FBSDE

$$\begin{cases} 2\,\eta\,\mathrm{d}\nu_t^\star &= \left(-A_t\,F_t + \beta\,I_t + (\phi + c\,\beta)\,(Y_t + Q_t) + c\,\beta\,Z_t\right)\mathrm{d}t + \mathrm{d}M_t, \\ 2\,\eta\,\nu_T^\star &= c\,\left(Y_T + Q_T\right)\,, \\ \\ \mathrm{d}Z_t &= \left(\beta\,(Z_t + Y_t + Q_t) - G_t\,F_t\right)\mathrm{d}t + \mathrm{d}N_t, \\ Z_T &= 0\,, \\ \\ \mathrm{d}I_t &= \left(c\,\nu_t^\star - \beta\,I_t\right)\mathrm{d}t, \\ I_0 &= 0\,, \\ \\ \mathrm{d}Q_t &= \nu_t^\star\,\mathrm{d}t\,, \end{cases}$$

for some $\mathbb{F}$-martingales M and N such that $M_T, N_T \in L^2(\Omega)$.

Three Stages — Stage II

Proposition (Differential Ricatti Equation)

Let (a bunch of matrices)... Suppose there exists a solution P, which is an $\mathbb{R}^{2\times 2}$-valued C^1 function, to the DRE

$$P'(t) + P(t)\,B_{11} + P(t)\,B_{12}\,P(t) - B_{21} - B_{22}\,P(t) = 0\,, \quad P(T) = G$$

Define $\mathbb{R}^2$-valued processes ℓ, Ψ, and Φ in the following way:

$$\ell_t = e^{-\int_0^t (P(u)\,B_{12} - B_{22})\,\mathrm{d}u}\,\mathbb{E}\left[L - \int_t^T e^{\int_0^s (P(u)\,B_{12} - B_{22})\,\mathrm{d}u}\, b_s\,\mathrm{d}s \,\middle|\, \mathcal{F}_t \right] ,$$

$$\Phi_t = e^{\int_0^t (B_{12}\,P(u) + B_{11})\,\mathrm{d}u} \left(K + \int_0^t e^{-\int_0^s (B_{12}\,P(u) + B_{11})\,\mathrm{d}u}\, B_{12}\,\ell_s\,\mathrm{d}s \right) ,$$

and

$$\Psi(t) = P(t)\,\Phi_t + \ell_t\,.$$

Then (Φ, Ψ) is a solution to the FBSDE with

$$\Psi_t = \begin{pmatrix} \nu_t^\star \\ Z_t \end{pmatrix} , \quad \Phi_t = \begin{pmatrix} I_t \\ Q_t \end{pmatrix} .$$

Moreover, the DRE admits a unique solution.

Three Stages — Stage II

Proposition (No Transient Impact)

Assume $c = 0$. *The optimal hedging strategy in the CEX is*

$$\nu_t = P(t) \left(Q_0 \, \tilde{P}(0,t) + \int_0^t \tilde{P}(s,t)\, \ell_s \, \mathrm{d}s \right) + \ell_t \,,$$

where

$$\ell_t = \tfrac{1}{2\,\eta} \mathbb{E}\left[\int_t^T \tilde{P}(t,s)\,(A_s\,F_s - \phi\,Y_s)\,\mathrm{d}s \,\middle|\, \mathcal{F}_t \right] ,$$

and

$$P(t) = \sqrt{\tfrac{\phi}{2\,\eta}} \tanh\left(\sqrt{\tfrac{\phi}{2\,\eta}} (t - T) \right) \qquad \textit{and} \qquad \tilde{P}(s,t) = \frac{\cosh\left(\sqrt{\tfrac{\phi}{2\,\eta}} (t - T) \right)}{\cosh\left(\sqrt{\tfrac{\phi}{2\,\eta}} (s - T) \right)} .$$

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  - profitability $\gamma = \frac{\lambda\,\pi\,(v-\pi)}{2}$

Three Stages — Stage I

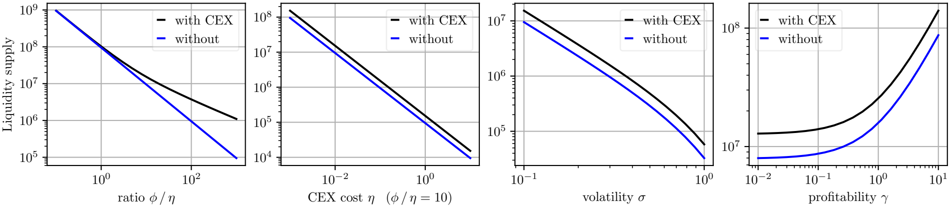

Figure: Equilibrium supply of liquidity as a function of model primitives. Default parameter values are: fee rate $\pi = 0.3\%$, volatility $\sigma = 0.1$, investment horizon $T = 1$, private signal $A = 0$, CEX trading cost $\eta = 0.01$, ratio $\beta = \phi/\eta = 10$, and profitability $\gamma = 0.2$.

Three Stages — Stage I

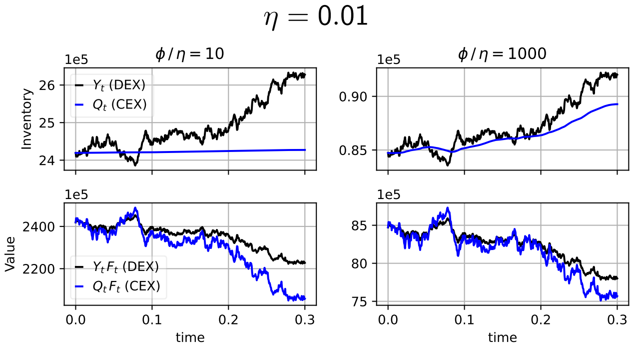

Figure: Sample path of the LP's reserves Y_t held in the DEX and the inventory Qt held in the CEX (top panels), together with their corresponding values expressed in units of the reference asset X (bottom panels). The left panels of each figure correspond to a ratio of risk aversion to trading costs $\beta = 10$, while the right panels correspond to $\beta = 10^3$. Other default parameter values are profitability $\gamma = 0.1$, fundamental volatility $\sigma = 0.2$, and investment horizon $T = 0.3$.

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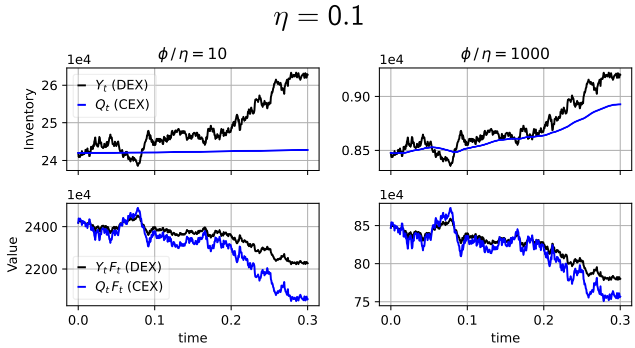

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Three Stages — Stage I

- When the LP executes her **optimal CEX strategy**, her change in wealth, measured in units of X, is

$$\underbrace{\int_0^T \Pi(F_t,\kappa^\star)\,dt}_{\text{fee revenue}} + \underbrace{2\,\kappa^\star\left(F_T^{1/2}-F_0^{1/2}\right)}_{\text{AMM position value change}} - \underbrace{\int_0^T Q_t^\star\,dF_t}_{\text{risk offsetting}} - \underbrace{\int_0^T \eta\,\nu_t^{\star\,2}\,dt}_{\text{CEX cost}},$$

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- When the LP **does not offset**, her change in wealth is

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Three Stages — Stage I

The expected change in the value of the LP's DEX liquidity position is

$$\mathbb{E}\left[2\,\kappa^{\star}\,(F_T^{1/2} - F_0^{1/2})\right] = F_0^{1/2}\left(e^{-\sigma^2\,T/8} - 1\right) < 0$$

— viability of DEX liquidity provision depends on whether stage-three fee revenue, adjusted by replication costs and the proceeds form risk offsetting, cover these adverse selection costs.

Three Stages — Stage I

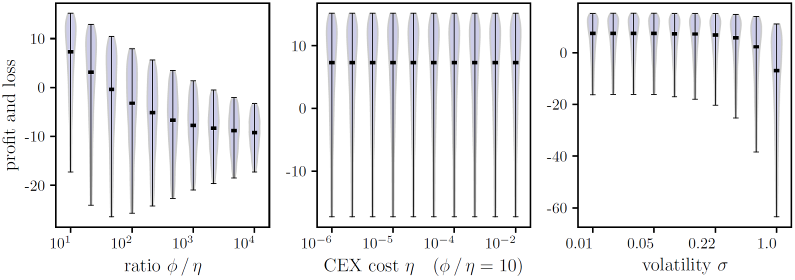
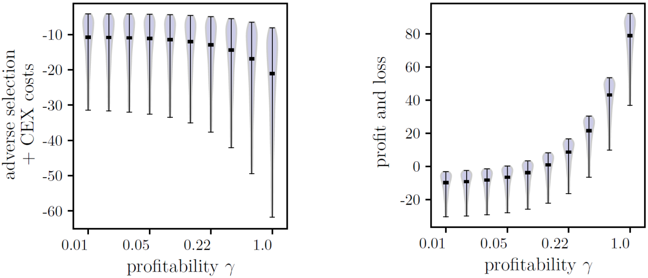


Figure: The distribution is obtained from 2000 market simulations, with the time interval discretised into 1000 steps. Default parameter values are $\sigma = 0.1$, $T = 1$, $A = 0$, $\eta = 0.01$, $\beta = 10$, and $\gamma = 0.25$.

Three Stages — Stage I



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Thank you....

...Questions & Comments?