From Micro-Foundations to Market Dynamics: A Structural Cointegration Model of Bitcoin Prices

Decentralized Finance & Crypto Workshop

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Motivation

- Bitcoin evolved from a digital experiment to a globally traded financial asset, **reaching institutional portfolios** (Corbet et al., 2022; Biais et al., 2023).
- **Controversial price dynamics:**
  - Speculative and sentiment-driven behavior (López-Cabarcos et al., 2021; Lin, 2021).
  - Emerging linkages with macro-financial fundamentals (Conlon and McGee, 2020; Umar et al., 2021).
- Literature focuses on short-run volatility and return predictability (Wang, Shen, and Li, 2022; Demir et al., 2021; Jareño et al., 2021)

  $\Longrightarrow$ **Long-run equilibrium structure largely unexplored.**
- **Key idea: model Bitcoin prices as equilibrium outcomes of investor demand and exogenous supply.**

Gaps in the Literature

- **Predominantly reduced-form evidence**
  - Focus on statistical properties, volatility, and return predictability
  - Limited connection to economic primitives(Wang, Shen, and Li, 2022; Peng et al., 2024; Jalal, Alon, and Paltrinieri, 2025)
- **Limited use of structural equilibrium models**
  - Few contributions derive Bitcoin prices from investor behavior and market clearing
  - Notable exceptions do not deliver estimable cointegrating relations (Pagnotta, 2022)
- **Protocol-driven supply largely ignored**
  - Bitcoin supply is deterministic and inelastic
  - Halving events induce discrete structural breaks (Rudd and Porter, 2025; Gezer, 2025)

Research questions

RQ1: What long-run equilibrium relationship links Bitcoin's market capitalization to macro financial and energy related fundamentals?

(Wang, Shen, and Li, 2022; Conlon and McGee, 2020; Umar et al., 2021)

RQ2: How do equilibrium elasticities evolve across halving regimes as the market matures?

(Wu, Lee, and Zhao, 2022; Gezer, 2025)

RQ3: How quickly does Bitcoin revert to its structural equilibrium after short run deviations, and does this speed of adjustment evolve over time?

(Biais et al., 2023; Kukacka and Kristoufek, 2023)

Contribution

- We propose a **semi-structural valuation framework** that:
  - Links Bitcoin's market capitalization to macro-financial fundamentals
  - Explicitly incorporates protocol-driven supply
- The framework yields:
  - A stable long-run equilibrium relation
  - A disciplined interpretation of short-run deviations
- A key innovation is the **halving-based segmentation**:
  - Each halving defines a distinct economic regime
  - Structural elasticities and adjustment speeds are allowed to evolve over time

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Theoretical framework

- Investor allocates wealth W_t among Bitcoin (B), equities (S), and gold (G).
- Preferences follow a **Cobb–Douglas** utility:

$$U = q_{B,t}^{\alpha_B} q_{S,t}^{\alpha_S} q_{G,t}^{\alpha_G}.$$

- Budget constraint:

$$p_{B,t} q_{B,t} + p_{S,t} q_{S,t} + p_{G,t} q_{G,t} = W_t.$$

- **Maximization** $\implies$ individual demand:

$$q_{B,t} = \alpha_B \frac{W_t}{p_{B,t}}.$$

- **Bitcoin supply $S_{B,t}$ is fixed by design** (Nakamoto, 2008; Badev and Chen, 2014)

Theoretical framework

2 Methodology and data

- **Equilibrium price (market clearing):**

$$S_{B,t} = q_{B,t} \implies \boxed{p^*_{B,t} = \frac{\alpha_B W_t}{S_{B,t}}.}$$

- **Wealth proxy (macro-financial wealth/liquidity)** and **Preference weight with observable taste shifters:**

$$W_t \propto p_{S,t}^{\beta_S}\, p_{G,t}^{\beta_G}, \qquad \alpha_{B,t} = \bar{\alpha}_B\, p_{E,t}^{\beta_E}\, r_t^{\beta_R}\, T_t^{\beta_T}.$$

$p_{E,t}$ proxies mining/energy conditions; r_t is the opportunity-cost channel; T_t captures adoption/attention.

Theoretical framework

- Collecting terms yields the **constant-elasticity equilibrium price**:

$$p^*_{B,t} = \beta_0 \, p_{S,t}^{\beta_S} \, p_{G,t}^{\beta_G} \, p_{E,t}^{\beta_E} \, r_t^{\beta_R} \, T_t^{\beta_T} \, S_{B,t}^{-1},$$

where $\beta_0 > 0$ absorbs $\bar{\alpha}_B$ and proportionality constants.

$p_{S,t}$ equity market (risk appetite);
$p_{G,t}$ gold price (safe haven);
$p_{E,t}$ energy cost (mining profitability);
r_t risk-free interest rate (opportunity cost);
T_t technology trend.

- Taking logs gives the **estimable long-run relation**:

$$\log p^*_{B,t} = \beta_0 + \beta_S \log p_{S,t} + \beta_G \log p_{G,t} + \beta_E \log p_{E,t} + \beta_R \log r_t + \beta_T T_t - \log S_{B,t} + \varepsilon_t.$$

Theoretical framework

- Considering **market capitalization**:

$$\boxed{\log p^{*}_{B,t} + \log S_{B,t} = \beta_0 + \beta_S \log p_{S,t} + \beta_G \log p_{G,t} + \beta_E \log p_{E,t} + \beta_R \log r_t + \beta_T T_t + \varepsilon_t.}$$

β_S, β_G diversification and substitution effects;
β_E energy cost channel;
β_R cost of capital;
β_5 secular trend (adoption);

Empirical design: challenges and solutions

1. **Halving regimes (structural breaks)**
   - Every four years, the **block reward halves**, reducing new BTC issuance.
   - Each halving defines a distinct **market regime** with new equilibrium conditions.
   - Models are estimated **separately by regime**, treating supply as **exogenous**.
2. **Cointegration within each regime**
   - Price and macro-financial variables are **nonstationary** $(I(1))$.
   - However, they may be **cointegrated** $\implies$ long-run equilibrium relation.
   - To capture both dynamics:
     - Use **Dynamic OLS (DOLS)** to estimate long-run elasticities.
     - Use **Error-Correction Model (ECM)** to model short-run adjustments.

Goal: Estimate long-run equilibrium and short-run adjustments within each regime.

Empirical design

1. **Segmentation by halving period**
   - The sample is divided into four regimes defined by Bitcoin's halving events.
   - Each regime reflects different mining incentives and market equilibria.
2. **Stationarity and cointegration tests**
   - **ADF** tests on levels and differences to identify $I(1)$ processes.
   - **Johansen** and **Engle–Granger** tests check for cointegration among variables.
3. **Model setup**
   - Target: **log free-float market capitalization** $y_t = \log(p_{B,t}) + \log(S_{B,t})$.
   - Regressors: equity $(p_{S,t})$, gold $(p_{G,t})$, energy $(p_{E,t})$, interest rate (r_t), and trend (T_t).

4. **Long-run equilibrium (DOLS)**
   - Estimate:

$$y_t = \beta_0 + \boldsymbol{\beta}' X_t + \sum_{k=-K}^{K} \boldsymbol{\Gamma}_k' \Delta X_{t-k} + u_t.$$

   - Leads and lags of ΔX_t correct for endogeneity and serial correlation.
   - Long-run coefficients $\boldsymbol{\beta}$ are interpretable as **structural elasticities**.
   - HAC covariance (5 lags) ensures robust inference.

Empirical design

5. **Short-run adjustment (ECM)**
   - Use the residuals $\hat{u}_{t-1}$ from DOLS as the equilibrium correction term:

$$\Delta y_t = \alpha\,\hat{u}_{t-1} + \sum_{j=0}^{P} \phi_j' \Delta X_{t-j} + \varepsilon_t, \qquad \hat{u}_{t-1} = y_{t-1} - (\hat{\beta}_0 + \hat{\boldsymbol{\beta}}' X_{t-1}).$$

   - α measures the **speed of adjustment** toward equilibrium.
   - A **negative and significant** α implies mean reversion toward the structural equilibrium.

Data

2 Methodology and data

- **Dataset:** Daily observations from **2012–2024**.
- **Segmentation:** The sample is divided into **four halving periods** reflecting Bitcoin's protocol-driven block reward schedule.
- **Variables and Sources:**

Symbol	Description	Source / Proxy
$p_{B,t}$	Bitcoin price (USD, log)	**CoinMetrics**
$S_{B,t}$	Circulating supply (BTC, log)	**CoinMetrics**
$p_{S,t}$	Equity market price	S&P 500 index — **Yahoo Finance**
$p_{G,t}$	Gold price	London Bullion Market (LBMA) — **Yahoo Finance**
$p_{E,t}$	Energy cost proxy	Henry Hub natural gas futures — **U.S. EIA**
r_t	Opportunity cost of capital	U.S. Treasury yield (10Y) — **Yahoo Finance**
T_t	Investor attention	**Google Trends (topic: "Bitcoin")**

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3 Results and discussion

Results: Stationarity and cointegration tests

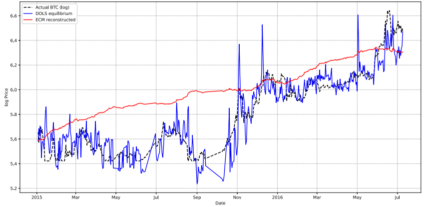
- All variables are **integrated of order one (I(1))**: levels nonstationary, differences stationary.
- **Johansen trace test:** confirms at least one long-run equilibrium per halving period.
- **Pairwise BTC–X tests:** mostly insignificant $\implies$ cointegration is **joint**, not pairwise.

Halving	1	2	3	4
Johansen rank	2	2	1	1
ADF (I(1) confirmed)	Yes	Yes	Yes	Yes
Cointegrated jointly?	Yes	Yes	Yes	Yes
Pairwise BTC–X p-value < 0.05	Few	Few	1 (trend)	None

Interpretation: Bitcoin's equilibrium is defined by a common stochastic trend involving multiple macro-financial variables.

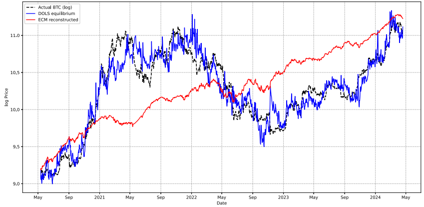
Reconstructed Equilibrium Prices across Halving Regimes

Halving 1 (2012–2016)

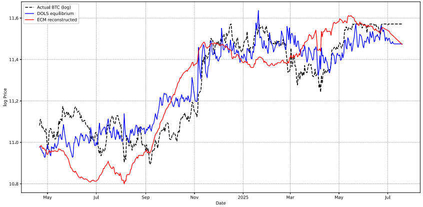


Halving 2 (2016–2020)

Halving 3 (2020–2024)



Halving 4 (2024–)



Results: Long-run equilibrium (DOLS)

- Bitcoin's equilibrium price responds to **equity markets**, **interest rates**, and **long-run adoption**.
- The estimated long-run elasticities evolve across halvings, revealing **structural transitions.**

Halving	S&P 500	UST (rates)	Gold	Energy	Trend
H1 (2012–16)	1.62***	0.21***	0.38**	-0.01	0.54***
H2 (2016–20)	4.44***	0.08†	0.40	-0.23†	0.69***
H3 (2020–24)	3.48***	-0.05***	-0.50†	-0.06†	0.40***
H4 (2024–)	0.90***	-0.74***	0.57**	0.10**	0.15***

Results: Long-run equilibrium (DOLS)

- **Early halvings (H1–H2) – speculative, liquidity-based asset**
  - Equilibrium dominated by **equity market conditions**.
  - Very large S&P elasticities ($\beta_S > 1$, up to 4.4).
  - Strong trend elasticity $\implies$ **adoption and attention drive valuation**.
  - Interest rates and energy largely irrelevant.
- **Later halvings (H3–H4) – macro-financially integrated instrument**
  - Interest-rate elasticity turns **negative and significant**.
  - Gold switches to stable positive co-movement.
  - Energy becomes a positive long-run anchor.
  - Trend elasticity declines monotonically.

Interpretation: equilibrium valuation shifts from speculative attention to macro-financial fundamentals.

Results: Short-run dynamics (ECM)

- The error-correction term (α) measures how fast deviations from equilibrium are closed.
- BTC returns mainly react to equity returns in the short run.
- **Speed of adjustment** rises over time $\implies$ markets close gaps faster.
- **Short-run co-movement** with equities strengthens post-2020.

Halving	α	Half-life (days)	R^2	ΔS&P Coeff.
H1	-0.0169	41	0.02	0.04
H2	-0.0180***	38	0.05	0.74***
H3	-0.0101**	68	0.12	1.19***
H4	-0.0453***	15	0.17	0.97***

Results: Short-run dynamics (ECM)

- **Early phase (H1–H2):** BTC behaves as a **high-beta asset**, tracking global liquidity and speculative wealth effects.
- **Transition (H3):** Start of macro anchoring—interest rates gain explanatory power, trend importance fades.
- **Mature phase (H4):** BTC integrates into the global financial cycle:
  - Negative rate elasticity (discount rate effect);
  - Positive gold and energy link (inflation and production cost channels);
  - Strongest error correction (market efficiency).

Interpretation: Bitcoin's equilibrium has evolved from adoption-driven to macro-financially grounded.
Warning: markets become more efficient, but not less risky.

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Conclusions and future work

- Bitcoin's price follows a **demand-driven equilibrium**, with supply exogenous and fixed in the short run by protocol design.
- DOLS results show a shift from **liquidity-driven** to **macro-financial anchoring**:
  - Early halvings: equity and trend dominate (speculative phase);
  - Later halvings: interest rates and energy costs gain relevance.
- ECM confirms a **faster correction** toward equilibrium, consistent with a maturing market.
- **Future work:** extend the framework to include
  - **short-term supply flexibility** driven by mining activity,
  - **volatility and regime-switching** effects,
  - and **energy–hash rate feedback loops**.

Financial implications

- **Diversification**
  - Early Bitcoin: idiosyncratic but unstable diversifier.
  - Mature Bitcoin: less orthogonal to macro risk factors.
- **Hedging properties**
  - Safe-haven role becomes **state-dependent**.
  - Performance tied to monetary and inflation regimes.
- **Interpretability**
  - Structural elasticities provide economic meaning to valuation.
  - Error-correction captures mispricing and market discipline.

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Thank you for listening!
Any questions?