Swaps and Options on Ethereum Gas Fees: A Framework for Risk Management in DeFi

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EIP-1559

The Ethereum transaction fee mechanism determines the price needed to be paid for each transaction to be added to the blockchain. The gas fee is denominated in gwei, or gigawei (1 gwei $= 10^{-9}$ETH).

The base fee $b[i]$ for block i is determined based on the base fee $b[i-1]$, size $s[i-1]$of previous block $i-1$ and target block size s^*according to the following rule

$$b[i] = b[i-1]\left(1 + \phi \frac{s[i-1] - s^*}{s[i-1]}\right) \tag{1}$$

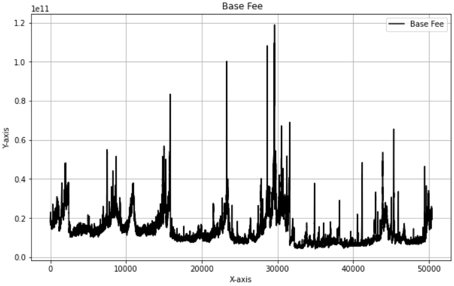
For blocks larger than the target block, the base fee increases; for blocks smaller than the target block, it decreases. The sensitivity of the base fee to the size of the previous block is determined by the adjustment parameter ϕ that is currently set to $\frac{1}{8}$ on Ethereum.

Problem Statement

While EIP-1559 improved fee estimation and partially stabilized bidding behavior, gas fees remain volatile due to endogenous congestion effects and application-specific spikes (e.g., NFT launches, DeFi liquidations). For context, the S&P 500's daily standard deviation of returns typically ranges from about 1.1% to 1.4% under normal market conditions and rises to roughly 2%–5% during periods of market stress. In contrast, the daily standard deviation of the base-fee log return can reach approximately 174%.

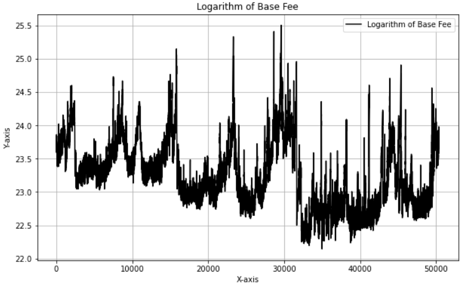
Data Analysis I

- The analysis is based on block level data, for block range from 12965000 to 15899264
- The time series exhibits cycles and spikes
- The data has a high positive excess kurtosis of 2057.6 and a significant positive skewness of 38.364.
- The base fee prices are significantly affected by the combination of supply and demand.



Data Analysis II

- Applying the logarithm transformation to the gas fee yields a time series with a distribution closer to normal, characterized by a mean of 24.386, a standard deviation of 1.004, skewness of -0.369, and excess kurtosis of -0.245.
- The log-transformed time series exhibits mean reversion
- The series can be modeled with Ornstein–Uhlenbeck process.



Swaps and Hedging

A **swap** is a financial derivative contract in which two parties agree to exchange streams of cash flows over a specified period, based on an agreed underlying reference (interest rates, currencies, commodities, or electricity prices). The main purpose of swaps is to manage price risk, stabilize cash flows, and improve budgeting.

A **swap**, with a **weighted average gas fee** as the underlying, is a contract in which two parties agree to exchange cash flows based on the difference between a fixed price and the weighted average gas price over a specified period. It is used to hedge against fluctuations in gas fees by locking in a predictable cost.

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Pricing Model for Swaps I

A swap contract is characterized by a trade date, a start date T_0, maturity date T_m, volume V , and fixed rate r_{fix} and reference floating rate r_{fl} The swap is a cash settlement instrument. The parties exchange payments on the maturity date of the contract. The amount M to be paid is calculated as the difference between the average base fee and the fixed rate, multiplied by the volume. If the difference is greater than zero, the protection buyer receives the amount; otherwise, the payment goes to the protection seller. Mathematically, the payment on the maturity date can be written as follows:

$$M = (r_{fl} - r_{fix})V \tag{2}$$

The floating rate is calculated as an sum of gas fee by blocks divided by number of blocks created between start and maturity date of the swap.

Pricing Model for Swaps II

The logarithm of base fee prices $X = ln(S)$ is assumed to follow an Ornstein-Uhlenbeck stochastic process:

$$dX_t = k(\alpha - X_t)dt + \sigma dW_t \tag{3}$$

The magnitude of speed adjustment $k > 0$ measures the speed of mean reversion to the long run mean log fee α. The second term in the equation (3) characterises the volatility of the process and W_t is standard Brownian motion. Under standard assumptions the dynamic of an Ornstein-Uhlenbeck process under the equivalent martingale measure can be rewritten as

$$dX_t = k(\alpha^* - X_t)dt + \sigma dW_t^* \tag{4}$$

where $\alpha^* = \alpha - \lambda$ and λ is the price of market risk and W_t^* is the Brownian motion under equivalent martingale measure.

Pricing Model for Swaps III

The condition distribution of X_t at time T is normal with mean and variance

$$E[X_T] = e^{-kT} X_0 + (1 - e^{-kT})\alpha^* \quad (5)$$

$$Var[X_T] = \frac{\sigma}{2k}(1 - e^{-2kT}) \quad (6)$$

Since $X = ln(S)$, the base fee at time T has the log-normal distribution with the same parameters. Assuming constant interest rate the future value of base fee at time T $F(S_T)$ is equals to the expectation of base of fee at time T

$$F(S_T) = exp(e^{-kT} \ln S_0 + (1 - e^{-kT})\alpha^* + \frac{\sigma}{4k}(1 - e^{-2kT})) \quad (7)$$

Pricing Model for Swaps IV

Under risk neutral measure a fair value of a swap contract on the inception date T_0 is equals to zero. So

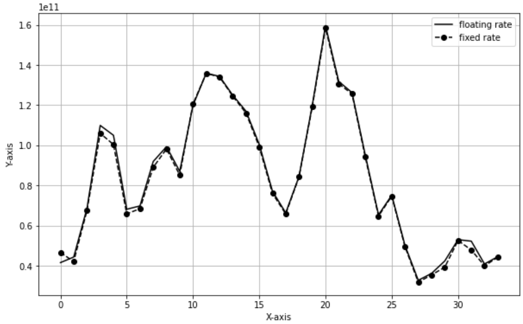
$$E((r_{fl} - r_{fix})V) = 0 \tag{8}$$

Hence

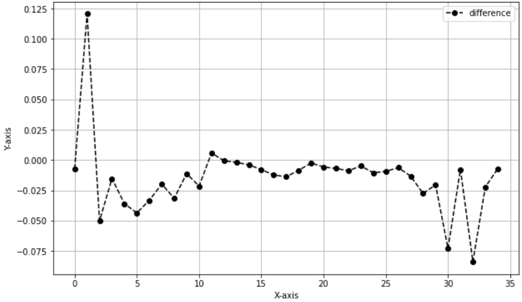
$$r_{fix} = E(r_{fl}) = \frac{1}{n}\sum_{i=1}^{n} F(S_i) \tag{9}$$

where n is the number of blocks between start date and maturity date of the swaps. Given this, we can infer that the fixed rate of the swap can be estimated as the average of the future values of the base fee.

Empirical Results



Fixed vs Floating rates



Relative Rate Spread

Takeaway

- Calibration window choice strongly affects model precision.
- Fixed rates are typically higher than floating rates, requiring dedicated hedging, especially against large spikes.
- As T increases, the forward rate converges to its long-term mean.
- Therefore, the influence of the current base fee declines and model accuracy may deteriorate for long maturities.

An European call option is a financial contract that gives the buyer the right, but not the obligation, to buy a specified underlying asset at a predetermined price on a specified expiration date. It is typically used when an investor expects the price of the underlying asset to rise, allowing them to benefit from upside price movements while limiting potential losses to the premium paid. The European option is characterized by the three parameters a trade date, a maturity date T, and a strike price K This instrument is settled in cash. At maturity date the payoff G of the European call option is:

$$G = (S_T - K)^+$$

where S_T denotes the gas fee price on the Ethereum network at time T and $x^+ = \max(x, 0)$ Hence, if the gas fee at maturity exceeds the strike price, the option holder receives the positive difference; otherwise, the payoff is zero

Pricing Model for Options I

Since gas fees are not directly tradable, the standard option pricing theory developed for complete markets cannot be applied directly. A replication portfolio can only be constructed using correlated tradable assets or indices, rather than the underlying gas fee itself. The effectiveness of such a replication strategy depends on the strength of the correlation between the tradable asset and the non-tradable gas fee. This leads to an incomplete market setting, in which perfect hedging is impossible and classical no-arbitrage pricing no longer applies. In this context, utility-based pricing methods become essential for valuing and managing the risk of these options.

Pricing Model for Options II

The gas fee price S_t follows an exponential Ornstein-Uhlenbeck process.

$$dS_t = S_t\Big[\Big(k(\alpha - \ln S_t) + \frac{\sigma^2}{2}\Big)dt + \sigma dW_t\Big] \tag{10}$$

Additionally, we assume that exists the trading asset Y_t correlated with gas fee prices. The price of it is a lognormal diffusion satisfying:

$$dY_t = bY_t dt + aY_t d\tilde{W}_t \tag{11}$$

The process $\tilde{W}_t$ and W_t are Brownian motions defined on the probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ where $\mathcal{F}_t$ is the augmented σ-algebra generated by $(\tilde{W}_u, W_u\ 0 \leq u \leq t)$. The Brownian motions are correlated with correlation $\rho \in (-1, 1)$. We also assume zero risk -free rate.

Pricing Model for Options III

The individual risk preference are defined according to utility function.

$$U(x) = -e^{-\gamma x} \quad \gamma > 0 \tag{12}$$

At the trading day of the option, the option seller starts with initial endowment m and rebalances his portfolio holdings by dynamically choosing the investment allocations. The amounts π_s^0 is put on the bank account and π_s is invested in a tradable risky asset. We assume that the risk-free interest rate is zero. Furthermore, no intermediate consumption or infusion of exogenous funds is permitted The current wealth $M_s = \pi_s^0 + \pi_s$ follows the control diffusion process:

$$dM_t = b\pi_t dt + a\pi_t d\tilde{W}_t \quad 0 < t < T \tag{13}$$

with $M_0 = m$. The set of all admissible control is denoted as $\mathcal{Z}$.
The call option's writer seeks to maximize expected utility of terminal wealth at time T denoted by u.

$$u(m, y, t) = \sup_{\mathcal{Z}} \mathbf{E}(-e^{-\gamma(M_T - G)} | M_t = m, S_t = s) \tag{14}$$

Pricing Model for Options IV

The indifference writer's price of the European claim $G = g(S_T)$, is defined as the function $h \equiv h(m, y, t)$, such that the investor is indifferent towards the following two scenarios: optimize the expected utility without employing the derivative and optimize it taking into account, on one hand, the liability $G = g(S_T)$ at expiration T, and on the other, the compensation $h(m, s, t)$ at time of inscription t Therefore,

$$\sup_{\mathcal{Z}} \mathbf{E}(-e^{-\gamma(M_T)}|M_t = m) = \sup_{\mathcal{Z}} \mathbf{E}(-e^{-\gamma(M_T+h-G)}|M_t = m, S_t = s) \qquad (15)$$

Pricing Model for Options V

The price of the option :

$$h(t,s) \approx \mathbf{E}_{\mathbb{Q}}(g(S_t)) + \frac{(\gamma(1-\rho^2)^2}{2}\,\mathrm{Var}_{\mathbb{Q}}(g(S_t)) \tag{16}$$

The optimal number n of shares of a correlated asset to be held in the portfolio is:

$$n = \rho\frac{\sigma S_t}{aY_t}h_s(t,s) \tag{17}$$

Empirical Finding

- Base gas fee levels on the Ethereum blockchain are driven primarily by overall transaction volume.
- According to data from the Dune Analytics platform, most transaction activity on Ethereum originates from: (Bitcoin, Ethereum,Stablecoins such as USDT and USDC GOLDEN, a digital asset representing ownership of physical gold on-chain
- Over time, the USD prices of these high-volume assets show a relatively weak correlation with Ethereum base gas fees in USD.
- In contrast, meme tokens and certain financial-service-oriented tokens exhibit low on-chain activity, demonstrate strong price–gas relationships
- Show correlations exceeding 75%
- Meme-token examples: PEPI, DOGE, GROKSTER, COCORO
- Financial-service-linked token examples:
  - RNB (rental-service protocol)
  - KUJI (yield-generating protocol token)
  - TAP (banking-bridge token)
  - WIN (native token of the WinCoin network)

Thank you!